Composite Statistical Method for Modeling Wind Gusts

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This paper discusses the application of three statistical methods in combination to model wind gusts for use in aircraft flight simulation. The approach combines principal components analysis, time-series analysis, and probability distribution methods to analyze and simulate wind gust components. Comparisons between wind gust components generated by the model and those measured onboard an aircraft show the model produces realistic gust components.

Nomenclature

a_i	= autoregressive coefficient
AR(p)	= autoregressive model of order p
ARIMA (p,d,q)	= autoregressive integrated moving average model of orders p and q with d differences
b_j	= moving average coefficient
$e(t)$	= random error
GLD	= generalized lambda distribution
MA(q)	= moving average model of order q
PCA	= principal components analysis
t	= time, s
U,V,W	= three components of wind gust, m/s
$Z(t)$	= time-series variable
ZB	= mean of time series

Introduction

A REALISTIC simulation of aircraft flight requires that all the control inputs and external forces be accurately modeled. One of the important external forces that must be modeled is the effect of wind gusts. Because of the random nature of wind gusts, realistic models of the horizontal and vertical gust components are difficult to obtain.

Several approaches to modeling gusts are available.[1] One approach is to use standard probability distributions to represent the number and velocity of gusts as discrete events. In the frequency domain, a model, such as von Kármán's, can be used to describe the correlation and spectra of gusts. For use in the time domain, impulse response-functions of gusts can be transformed by convolution or the inverse Fourier transform. Alternatively, filtered white noise can be shaped to the von Kármán spectrum, although pilots report the resulting simulation as bland.[1]

In this paper the random nature of wind gusts is tackled directly in the time domain with a combination of three statistical tools. The three methods are applied in sequence both to analyze the gust components and to produce a model of the components. The three methods have been available in the literature for a number of years, but both the application and the combination of these methods are novel. After a model of wind gusts has been produced with these methods, the model can be used to generate gust components useful for input to aircraft flight simulations.

Received July 1, 1985; presented as Paper 85-1750 at the AIAA Flight Simulation Technologies Conference, St. Louis, MO, July 22-24, 1985; revision received Oct. 22, 1985. This paper is declared a work of the U.S. Government and therefore is in the public domain.

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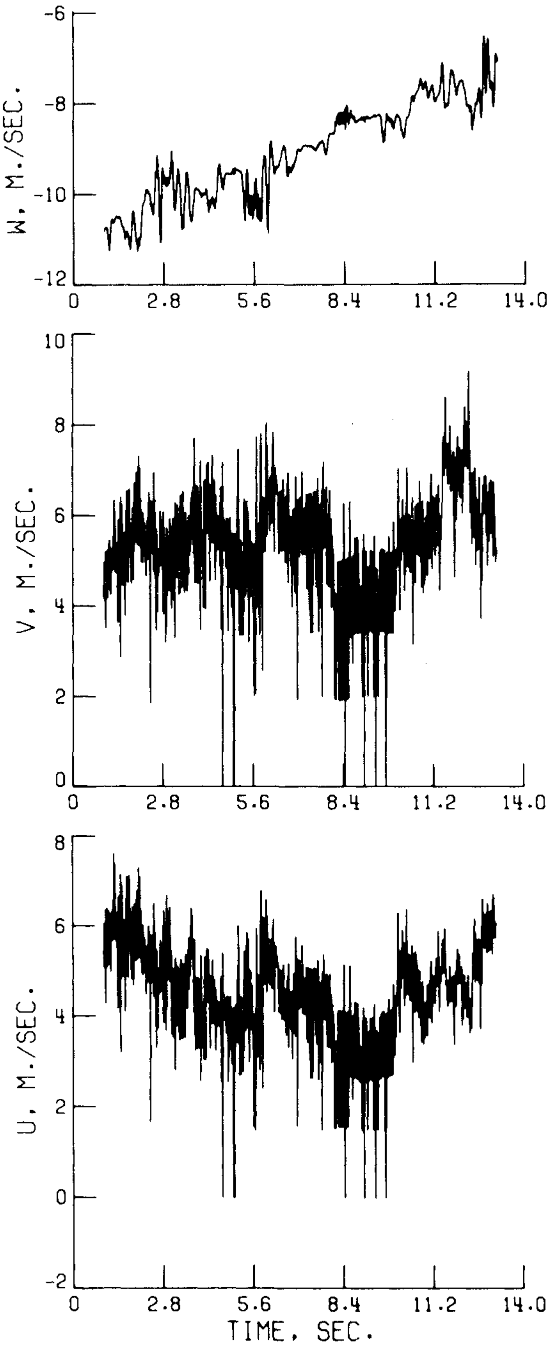

Fig. 1 Measured wind gust components.

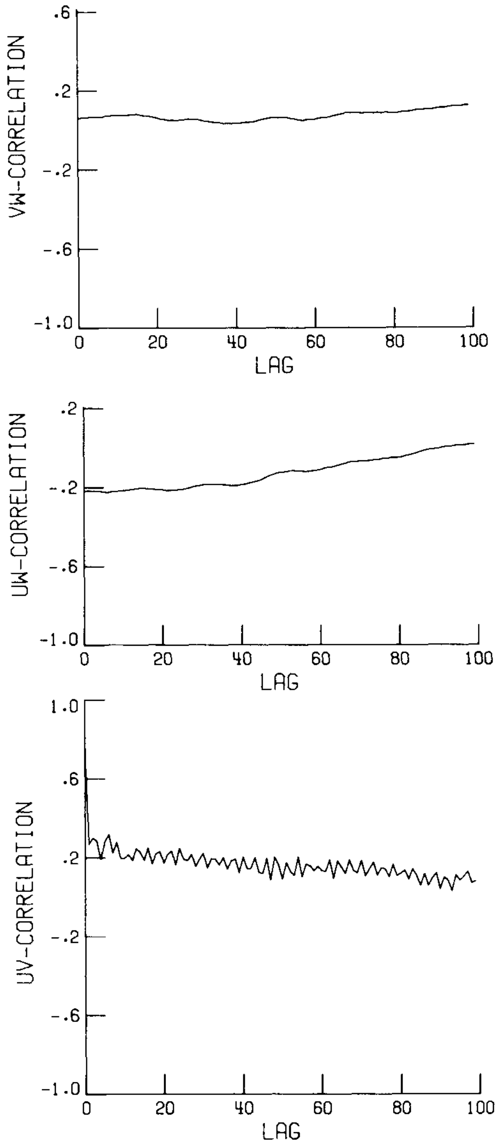

Fig. 2 Cross correlations of measured gust components.

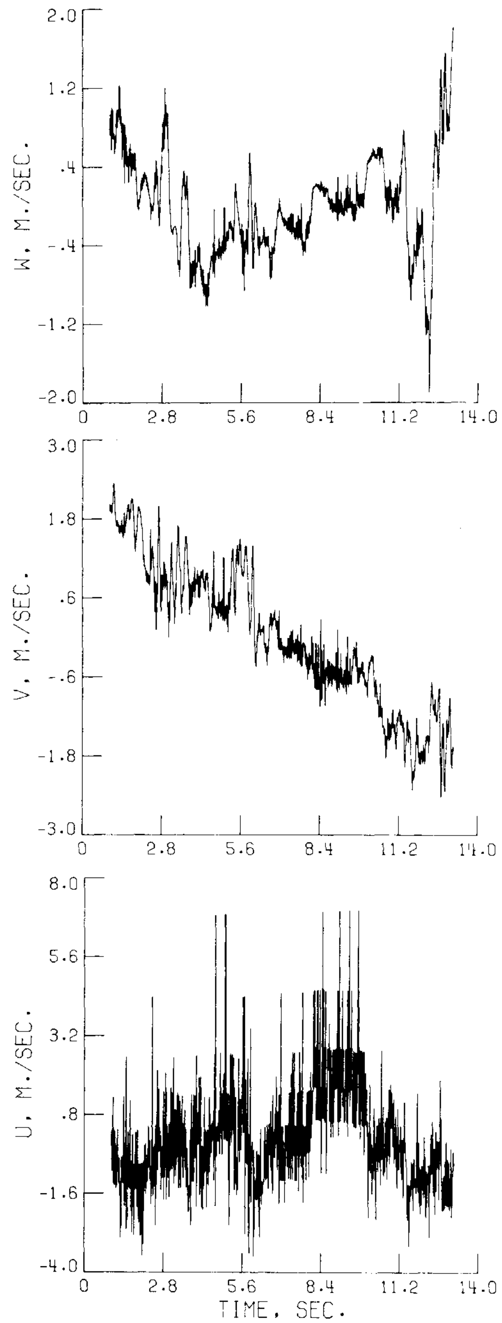

Fig. 3 Principal components of measured wind gusts.

Method and Approach

In this section, the three statistical techniques applied to wind gust data are described. The techniques are presented in the order in which they are applied to the data.

It is assumed that the wind gust measurements consist of the three components of wind in a three-dimensional Cartesian coordinate system (U, V, W) where U and V are horizontal axes and W is vertical. In general, the axes of this system will not be aligned with the wind direction; for this reason, any two of the three measured components will be correlated. If the components were not correlated, then each component could be analyzed separately.

The first task is, therefore, to transform to an axis system in which the measured components are uncorrelated. This is accomplished using principal components analysis (PCA). With PCA,[2] the 3×3 covariance matrix of all the measurements is first calculated. Then the three eigenvalues and corresponding eigenvectors of the covariance matrix are calculated. The eigenvectors are used to form a 3×3 matrix, which transforms the three-component gust vectors to a new axis system called the principal component axes. In the new axes, the axis corresponding to the largest eigenvalue and associated eigenvector is called the first principal component. The largest eigenvalue is the variance of the transformed measurement component along the first principal component axis. The second and third principal components are similarly defined by the eigenvectors associated with the second- and third-largest eigenvalues, and the eigenvalues are the variances of the transformed measurement components along these two axes. Essentially, PCA transforms to a new set of orthogonal axes aligned with the major dispersion of the measurements; in doing this, the new components become uncorrelated.

Given a set of gust measurement vectors whose components are uncorrelated, it is assumed that each of the three sets of (transformed) components defines a time series (set of measurements across time). Therefore, each component series can be analyzed separately using standard time-series analysis techniques. The model chosen to represent each component series is an autoregressive integrated moving average (ARIMA)[3] model of orders p,d,q:

$$Z(t) = ZB + \sum_{i=1}^{p+d} a_i Z(t-i) + \sum_{j=1}^{q} b_j e(t-j) + e(t) \qquad (1)$$

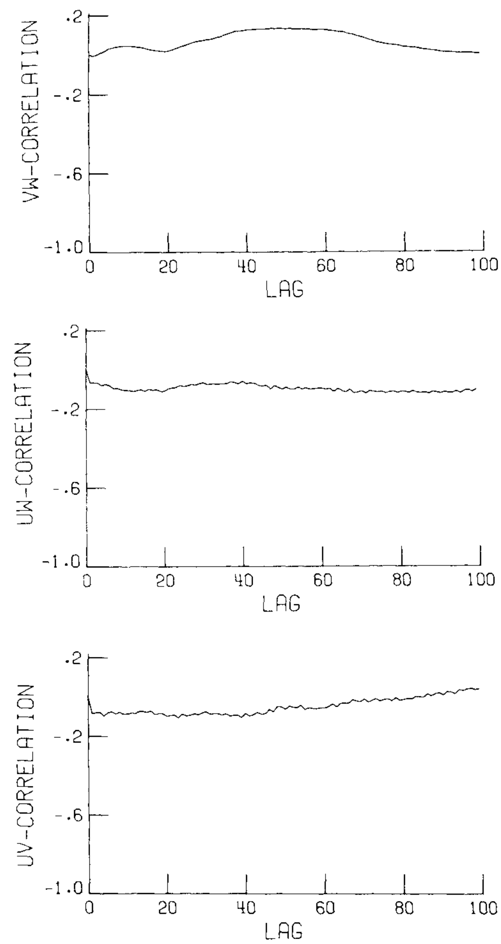

Fig. 4 Cross correlations of principal components of measured wind gusts.

In this equation, $Z(t)$ corresponds to one of the three transformed gust components (U,V, or W), and ZB is the mean value. The parameter d indicates the number of the times the original series was differenced; summation, the inverse of differencing, constitutes the "integrated" part of the model. The autoregressive model of order $p[\mathrm{AR}(p)]$ is given by the first summation that indicates that the series is a function of past values. The moving average of order $q[\mathrm{MA}(q)]$ is given by the second summation and third term, where $e(t)$ is a random input.

The random error $e(t)$ and constant coefficients a_i and b_j are unknowns that must be determined. The approach taken here is to estimate the coefficients and the statistics of the random error using standard Box-Jenkins techniques[3] for an ARIMA (p,d,q) model. With this approach the user has the freedom to choose the number of differences (d) and the orders (p,q) of the AR and MA models he considers most appropriate. The number of differences is chosen to remove linear and higher-order trends; p and q are chosen to be small integers but large enough to provide a reasonable fit.

The Box-Jenkins approach assumes that the random error is normally distributed with zero mean and constant variance. Normally distributed random values also have zero skewness and a kurtosis (peakedness) value of 3. One of the objectives of this study is to determine if a non-normal distribution better represents the random error for use in simulating the wind gust. To accomplish this, the first four statistical moments (mean, variance, skewness, and kurtosis) of the random error (residuals) found by the Box-Jenkins analysis are calculated. Then the generalized lambda distribution (GLD) of Ramberg and Schmeiser[4] is fit to the moments. The GLD is an analytical function that represents

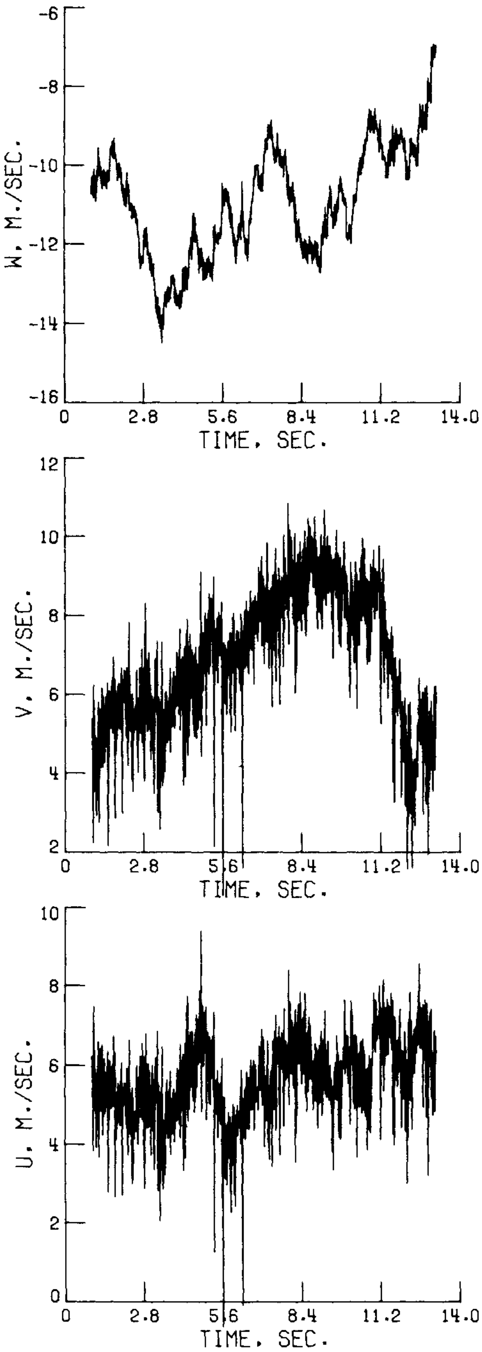

Fig. 5 Gust components simulated with GLD-generated random input.

the inverse cumulative distribution function of a probability distribution. Distributions covering a wide range of skewness and kurtosis values, including that of the normal distribution, can be represented by proper selection of the GLD parameters. Given a GLD fit to the moments of the random error, the GLD can be used to generate values of the random error for input to the ARIMA model in order to simulate the wind gusts.

Simulated wind gusts are generated by reversing the operations of the three statistical analysis steps. First, a sequence of random error values are generated using either the GLD or a normal random number generator. These values are combined with the AR and MA parameters in the ARIMA model [Eq. (1)] to generate a sequence of gust components in the principal components coordinates. Because three gust components are needed, three distinct sets of random error values are input to three different ARIMA models. The three sets of PCA gust components are combined to form a sequence of three-dimensional gust vectors over time. These vectors are then transformed back to the original gust coordinate system by applying in the inverse of the 3×3 PCA transformation matrix.

Results and Discussion

The data used in this study were gathered during flights of a NASA F-106B into severe storm centers. These data consist of 1000 measurements of three Cartesian components (two horizontal, one vertical) of the wind gust velocities adjusted for the aircraft motion. Several sets of measurements have been analyzed; the results of only one set will be presented since it is representative of all the results obtained.

Figure 1 is a plot of the gust components over time; the horizontal components consist of very noisy periodic trends. The vertical component contains an increasing linear trend, indicating strengthening updrafts encountered by the aircraft as it approaches the storm center. The trends in the components indicate that all three components are nonstationary. Figure 2 is a plot of the cross correlations of the gust components. All the cross correlations, such as those of U and V, are nonzero, suggesting misalignment between the axes and wind direction.

The PCA transformation was calculated from the covariance matrix of this data. The results of transforming the data to the principal components axes are shown in Fig. 3. It can be seen that the first principal component (U) contains the greatest variation and that the third (W) contains the least. Furthermore, Fig. 4 shows that all the cross correlations have been reduced; in particular, the cross correlations at zero lag are zero as prescribed by the PCA.

The Box-Jenkins algorithms used in this study are found in the IMSL software library.[5] Application of this software indicated that differencing the data once was sufficient to remove major trends. Box and Jenkins[3] also gives criteria (p. 34, 65) for determining the AR and MA orders of the model. For the three principal components of the wind gusts, these criteria indicated that p should be between 1 and 5 and q between 0 and 4. However, in practice, either the MA algorithm did not converge for $q>2$ or the time-series

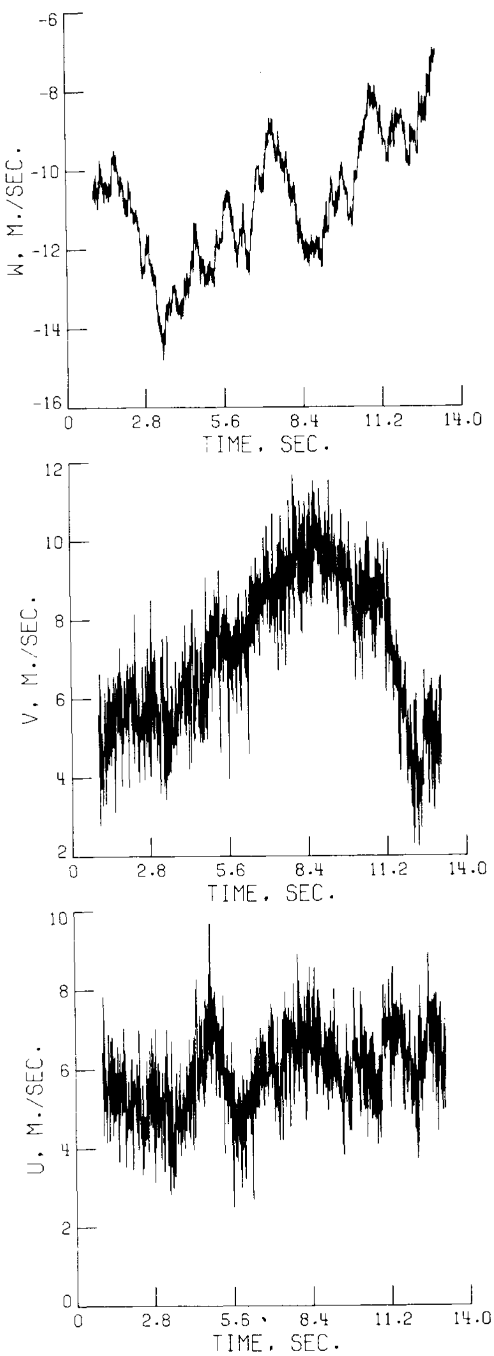

Fig. 6 Gust components simulated with normal-generated random input.

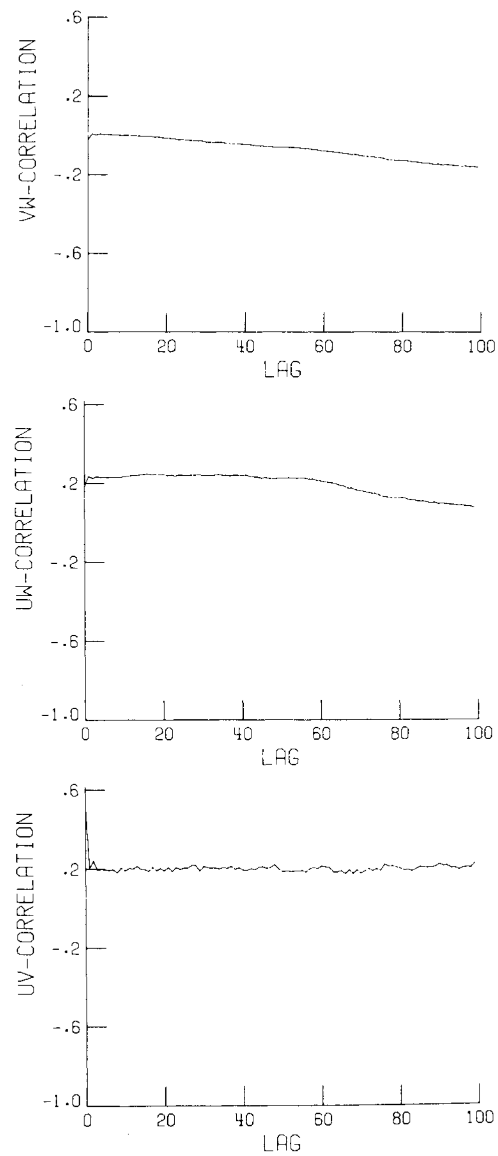

Fig. 7 Cross correlations of normal-generated wind gust components.

generated from higher-order AR models was too large in magnitude.

Experimentation with values of p and q indicated that the ARIMA (2,1,1) model gave an adequate representation of all three components. Analysis of the resulting residuals indicated that the residual distribution had skewness between 0.2 and 1.0 and kurtosis between 5.3 and 6.7. These values suggest that the GLD would be a better representation than would the normal distribution (skewness $=0$, kurtosis$=3$). The GLD parameters were estimated from the calculated moments with software developed at Langley Research Center.

Wind gusts were generated by inputting either GLD or normal random values into the ARIMA equations and transforming the ARIMA output with the PCA inverse transformation to the gust axes. Figures 5 and 6 present a comparison of the gusts generated with GLD and normal random values; except for slightly larger variation in the GLD-generated gusts, the two sets are very similar. The larger variation in GLD-generated gusts is evidently due to the positive skew in the distribution. Comparison of autocorrelations and cross correlations of the two sets with those of the original gusts indicate that the normal-generated gusts are more similar to the original gusts. Comparison of cross correlations of Fig. 2 and 7 show significant differences in magnitudes of both the UW and VW cross correlations. The differences are apparently due to the use of a low-order ARIMA model and the fact that the PCA transformation removes only the zero-lag cross correlations. Some of these differences can be reduced by increasing the ARIMA order.

Concluding Remarks

A method of modeling wind gusts that combines techniques from principal components analysis, time-series analysis, and probability distribution representation has been presented. The results of applying this method to gust measurements indicate that the method may be useful for simulating wind gusts. The PCA transformation removed most of the cross correlations between components in preparation for the Box-Jenkins time-series analysis. The fitted ARIMA model provides a good representation of the components if appropriate AR and MA orders are chosen. However, based on the data sets examined, there appears to be no substantial advantage to representing the random error with the GLD. The major difficulty in this application is the nonstationarity of wind gust measurements, because the ARIMA model applies only to stationary data. The approach taken here to remove nonstationary effects by differencing the data appeared to be adequate. However, removal of seasonal (periodic) effects was not attempted since available software for generating time series does not model these effects. The presented method is an alternative to current methods of generating wind gusts for aircraft flight simulation studies.

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